CS325 Artificial Intelligence
Ch 14b – Probabilistic Inference

Cengiz Günay

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Inference tasks

Simple queries: compute posterior marginal $P(X_i|\mathbf{E}=e)$

Conjunctive queries:

$$P(X_i, X_j|\mathbf{E}=e) = P(X_i|\mathbf{E}=e)P(X_j|X_i, \mathbf{E}=e)$$

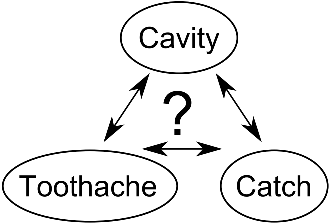
Optimal decisions: decision networks include utility information; probabilistic inference required for $P(outcome|action, evidence)$

Value of information: which evidence to seek next?

Sensitivity analysis: which probability values are most critical?

Explanation: why do I need a new starter motor?

With no dependency information, we need 2^n entries in joint dist.:



	toothache		¬ *toothache*	
	catch	¬ *catch*	*catch*	¬ *catch*
cavity	.108	.012	.072	.008
¬ *cavity*	.016	.064	.144	.576

Inference by Enumeration

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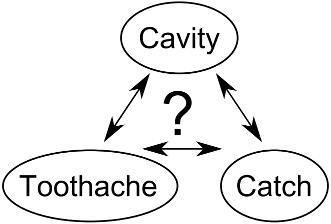

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
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For any proposition ϕ, sum the events where it is true:
$P(\phi) = \sum_{w:w \models \phi} P(w)$

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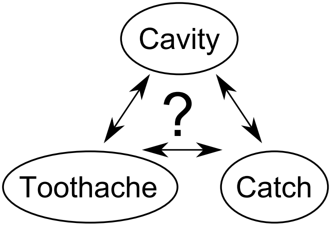

	toothache		$\neg$ *toothache*	
	catch	$\neg$ *catch*	*catch*	$\neg$ *catch*
cavity	**.108**	**.012**	**.072**	**.008**
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For any proposition ϕ, sum the events where it is true:

$P(\phi) = \sum_{w:w \models \phi} P(w)$

$P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

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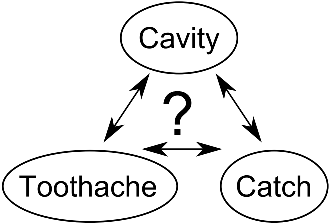

	toothache		$\neg$ *toothache*	
	catch	$\neg$ *catch*	*catch*	$\neg$ *catch*
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$P(cavity \vee toothache) = ?$

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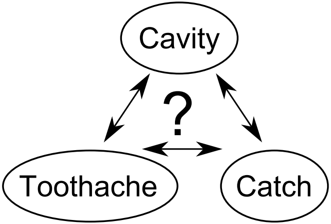

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	catch	¬ *catch*	*catch*	¬ *catch*
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For any proposition ϕ, sum the events where it is true:

$P(\phi) = \sum_{w : w \models \phi} P(w)$

$P(cavity \vee toothache) =$

$0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064 = 0.28$

Inference by Enumeration

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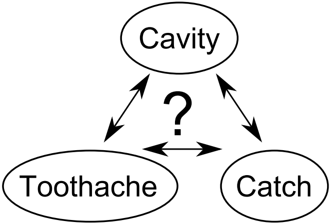

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cavity	**.108**	**.012**	**.072**	**.008**
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Can also compute conditional probabilities:

$$P(\neg cavity | toothache) = ?$$

Inference by Enumeration

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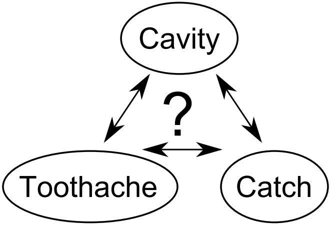

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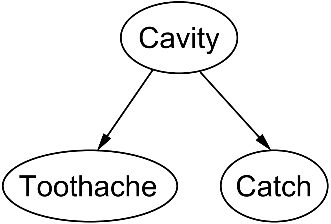
cavity	**.108**	**.012**	**.072**	**.008**
¬ *cavity*	**.016**	**.064**	**.144**	**.576**

Can also compute conditional probabilities:

$$P(\neg cavity|toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)}$$
$$= \frac{0.016 + 0.064}{0.108 + 0.012 + 0.016 + 0.064} = 0.4$$

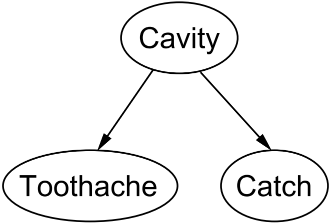
Joint probability with known dependencies



$$P(tootache, catch, cavity) = P(tootache|cavity) P(catch|cavity)P(cavity)$$

In general,
$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i|\mathrm{parents}(X_i))$

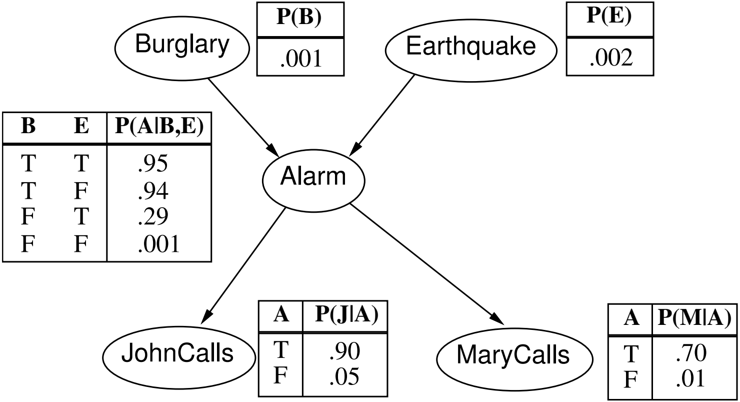
Joint probability with known dependencies



$$
\begin{aligned}
P(tootache, catch, cavity) &= P(tootache|cavity) \\
&\quad P(catch|cavity)P(cavity)
\end{aligned}
$$

In general,
$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i|\text{parents}(X_i))$

Burglary or Earthquake: inference from joint



P(B)
.001

P(E)
.002

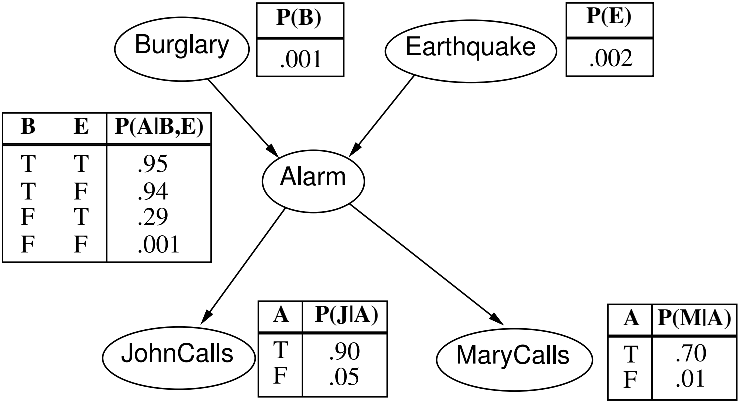
B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

$$P(j \wedge m \wedge a \wedge \neg b \wedge \neg e) \quad = \quad ?$$

Burglary or Earthquake: inference from joint



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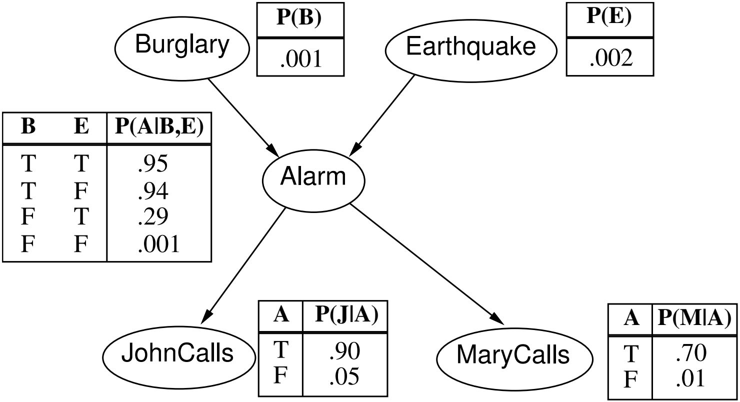
$$
\begin{aligned}
P(j \wedge m \wedge a \wedge \neg b \wedge \neg e) &= P(j|a)P(m|a)P(a|\neg b, \neg e)P(\neg b)P(\neg e) \\
&= 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 \\
&\approx 0.00063
\end{aligned}
$$

Burglary or Earthquake: inference by enumeration



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$$
\begin{aligned}
& P(B|j,m) \\
& = P(B,j,m)/P(j,m) \\
& = \alpha P(B,j,m) \\
& = \alpha \textstyle\sum_e \sum_a P(B,j,m,e,a)
\end{aligned}
$$

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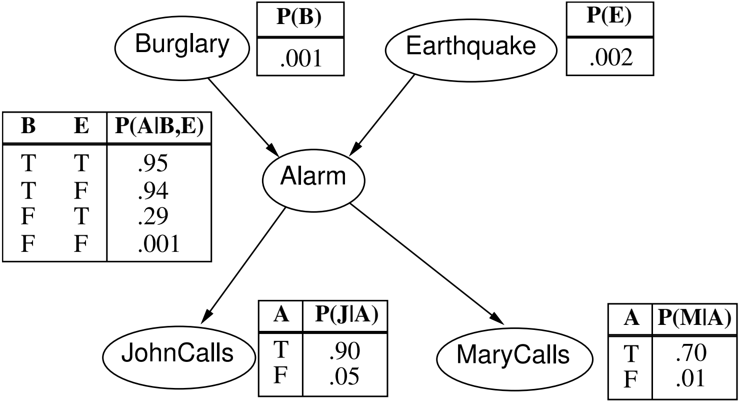

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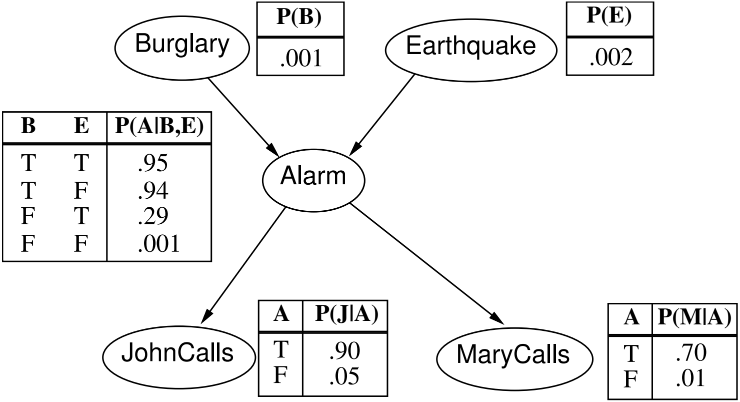
F	.01

$$P(B|j,m)$$
$$= \alpha \sum_e \sum_a P(B)P(e)P(a|B,e)P(j|a)P(m|a)$$
$$= \alpha \, P(B) \sum_e P(e) \sum_a P(a|B,e)P(j|a)P(m|a)$$

Burglary or Earthquake: inference by enumeration



- joining and elimination

What if we cannot infer exactly?

- Exact inference is expensive
- What else can we do?
- Observe random events and record outcomes to approximate probabilities
- Also called a Monte Carlo method
- With ∞ samples, it is *consistent*
- Rejection sampling: for rare events
- Likelihood weighing: to avoid inconsistency
- Gibbs sampling: random walk through state space
- Monty Hall letter

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