CS325 Artificial Intelligence
Ch. 7, 8, 9 – Logic, Knowledge, and Inference

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Spring 2013

Is Logic Overrated?

We did so far:

- Intelligent agents
- Problem Solving
- Probability
- Machine Learning

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Did we forget "thinking rationally?"
An agent needs logic for:

- To represent a model of the world
- And to reason about it

Entry/Exit Surveys

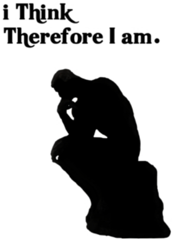
Exit survey: Unsupervised Learning

- What changed in your understanding?
- Any new suggestions on where would ***you*** use it?

Entry survey: Logic (0.25 points of final grade)

- What language would you use to represent logic?
- How would you make an agent reason?

It's been a while since Aristotle, do we still need formal logic?



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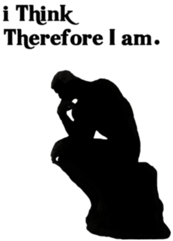

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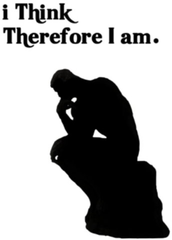


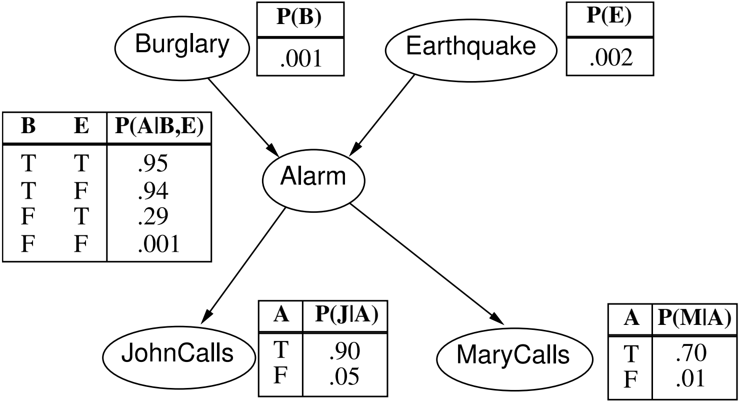
- Our society is based on logic: we take it for granted.

In this class, we'll learn the tools of logic for **representation** and **inference**:

- Propositional logic
- First-order logic

The Simplest: Propositional Logic

Remember?



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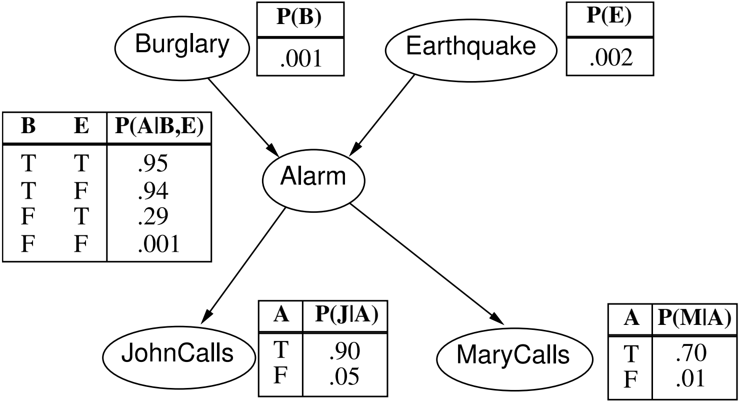

- $(E \vee B) \Rightarrow A$, Correct?

The Simplest: Propositional Logic

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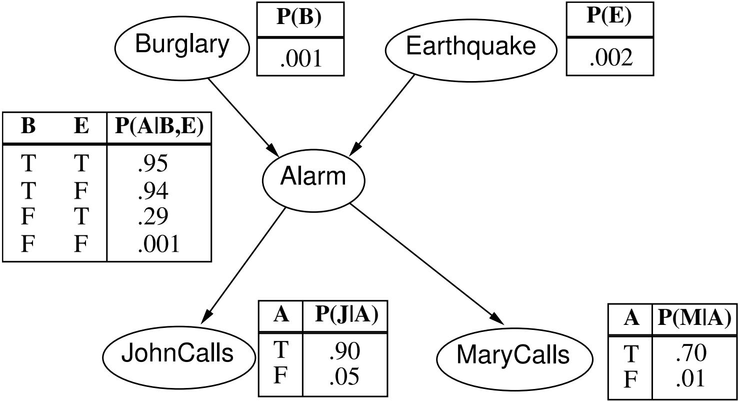

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- $A \Rightarrow (J \wedge M)$?

The Simplest: Propositional Logic

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Propositional Logic Operators Cheat Sheet

$\wedge$	And
$\vee$	Or
$\neg$	Negation
$()$	Grouping
$\Rightarrow$	Implies
$\Leftrightarrow$	Equivalence

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Propositional Logic Operators Cheat Sheet

$\wedge$	And
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$\Leftrightarrow$	Equivalence

Model of the world represented as: $\{B : \mathrm{True}, E : \mathrm{False}, \ldots\}$

Can You Handle the Truth Tables?

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
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Can You Handle the Truth Tables?

P	Q	$\neg P$	$P \land Q$	$P \lor Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
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- Mostly consistent with English meanings, except?

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Question: E: 5 is even, S: the earth goes around the sun

- $E \Rightarrow S$: True or False?

P	Q	$\neg$P	P $\wedge$ Q	P $\vee$ Q	P $\Rightarrow$ Q	P $\Leftrightarrow$ Q
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Question: E: 5 is even, S: the earth goes around the sun

- $E \Rightarrow S$: True or False?
- $\neg E \Rightarrow \neg S$: True or False?

Let's Put Truth Tables to Use

P	Q	$P \wedge (P \Rightarrow Q)$	$\neg(\neg P \vee \neg Q)$	$P \wedge (P \Rightarrow Q) \Leftrightarrow \neg(\neg P \vee \neg Q)$
False	False			
False	True			
True	False			
True	True			

Let's Put Truth Tables to Use

P	Q	$P \wedge (P \Rightarrow Q)$	$\neg(\neg P \vee \neg Q)$	$P \wedge (P \Rightarrow Q) \Leftrightarrow \neg(\neg P \vee \neg Q)$
False	False			Yes
False	True			Yes
True	False			Yes
True	True	Yes	Yes	Yes

Trick:

$$\neg(\neg P \vee \neg Q) \Rightarrow P \wedge Q$$

What we know to be True:

- $(E \vee B) \Rightarrow A$
- $A \Rightarrow (J \wedge M)$
- B

World Representation

What we know to be True:

- $(E \vee B) \Rightarrow A$
- $A \Rightarrow (J \wedge M)$
- B

Can we infer?

T	F	?	
			E
			B
			A
			J
			M

World Representation

What we know to be True:

- $(E \vee B) \Rightarrow A$
- $A \Rightarrow (J \wedge M)$
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Can we infer?

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Valid: Always true.

Satisfiable: Possible to be true.

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Validity and Satisfiability

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V	S	U	
			P
			$P \vee \neg P$
			$P \wedge \neg P$
			$P \vee Q \vee (P \Leftrightarrow Q)$
			$(Q \Rightarrow P) \vee (P \Rightarrow Q)$
			$(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party}) \Rightarrow (\text{Food} \wedge \text{Drinks} \Rightarrow \text{Party})$

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Valid: Always true.

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X			$(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party}) \Rightarrow (\text{Food} \wedge \text{Drinks} \Rightarrow \text{Party})$

Propositional Logic: Limitations?

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1. Only true and false propositions, no objects. Therefore no relations between objects
2. No uncertainty (except totally unknown entities)
3. No general statements like ALL or ANY
   Cumbersome for large domains.

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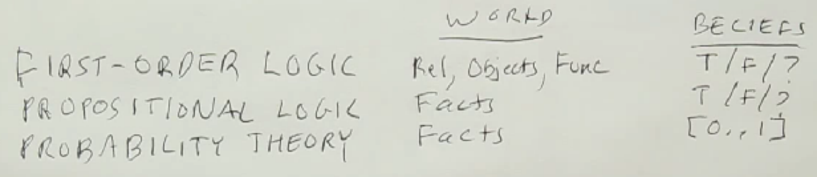
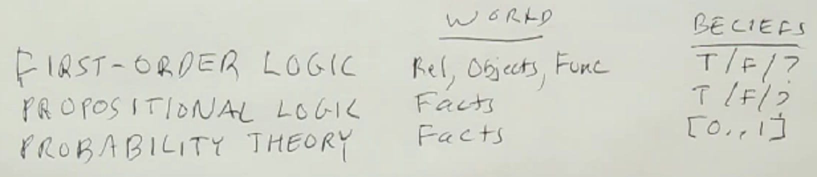
Next: **First Order Logic (FOL)**, fixes 1 & 3

First Order Logic



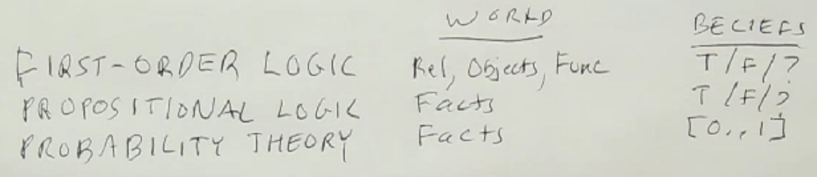
First Order Logic



Can also compare in terms of representation type:

1. Atomic: facts

First Order Logic



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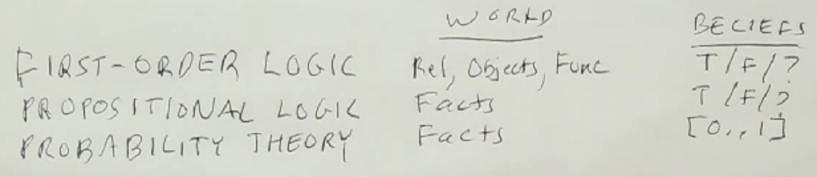

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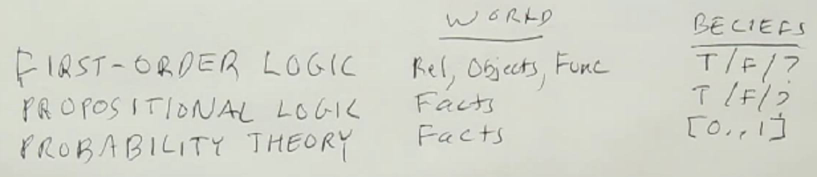

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FOL World Model

What was the model in propositional logic?

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$\{P: \text{True},\ Q: \text{False},\ \ldots\}$

FOL World Model

What was the model in propositional logic?
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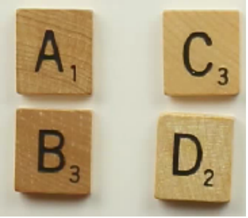
Let's represent these objects in First Order Logic:

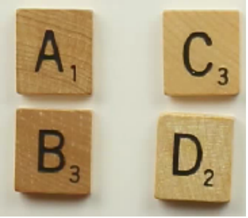


Constants: $\{A, B, C, D, 1, 2, 3\}$

Relations: above: $\{[A, B], [C, D], \ldots)$,
vowel: $\{[A]\}$
rainy: $\{\}$

Functions: numberof: $\{A \to 1, B \to 3, \ldots)$

FOL Syntax



Sentences
vowel(A)
above(A,B)
$2 = 2$

Terms
constants: $A, B, 2$
variables: x, y
func: numberof(A)

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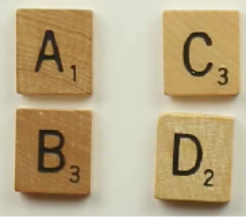

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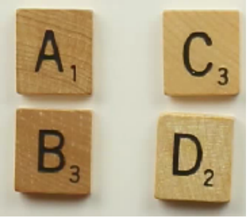

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above(A,B)
$2 = 2$

Terms
constants: $A, B, 2$
variables: x, y
func: numberof(A)

Operators: $\vee \wedge \neg \Rightarrow \Leftrightarrow ()$

Quantifiers: $\forall x \; \exists y$

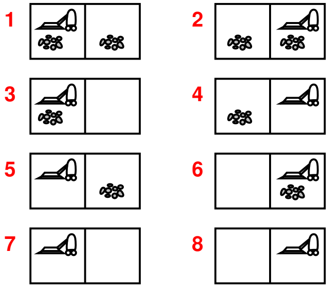
$\forall x \, \text{vowel}(A) \Rightarrow \text{numberof}(x) = 1$

$\exists x \, \text{numberof}(x) = 2$

Note: Default is $\forall$.

FOL in Action

Remember the 2-location vacuum world?



Constants: $A, B, V, D1, D2$

Relations: Loc, Vacuum, Dirt, At(o, l)

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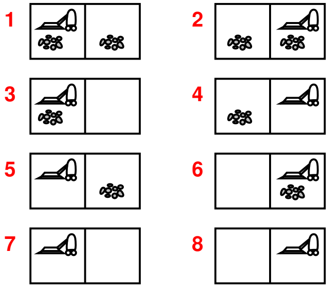

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1. Vacuum is at location A:

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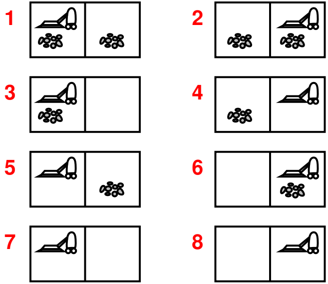

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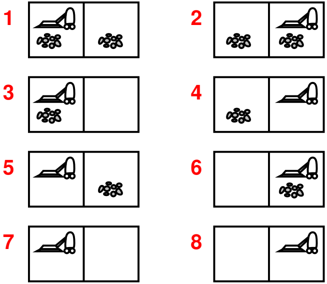

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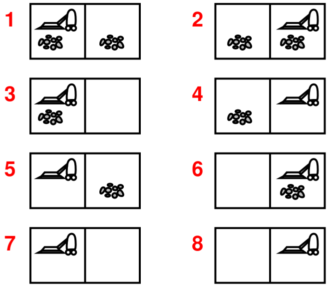
2. World is clean:

Remember the 2-location vacuum world?



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Say:

1. Vacuum is at location A: $\text{At}(V, A)$
2. World is clean:
   $\forall d\, \forall l\, \text{Dirt}(d) \wedge \text{Loc}(l) \Rightarrow \neg\text{At}(d, l)$
3. Vaccum is at dirty location

FOL in Action

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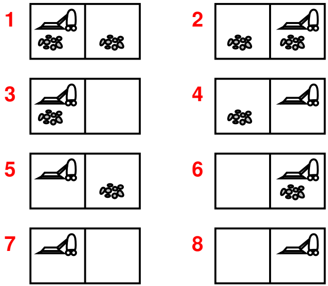

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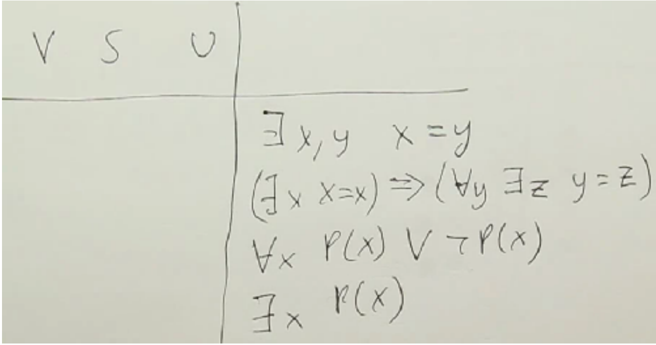
FOL Example



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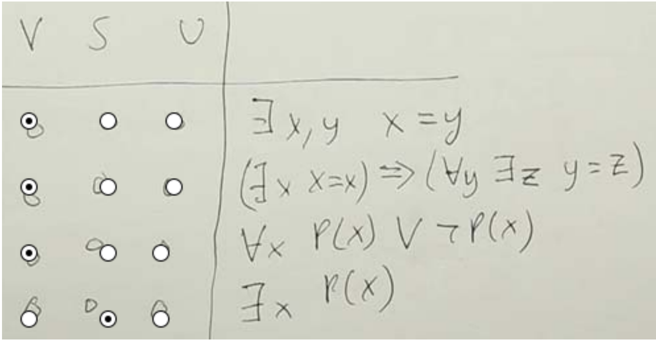

Exit Survey

Exit survey: Logic

- Where would you use propositional vs. FOL?
- What is the importance of logic representation over what we saw earlier?